Channel Selective Adaption of PWM Frequencies for Undisturbed AM and FM Reception in Automobiles

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Abstract — This work deals with the adaption of PWM frequencies to a chosen AM or FM broadcast channel in a vehicular application. To do so, the radio passes the information of the current receive channel to EMI sources that utilize PWM signals. By this feedback loop, the switching harmonics can be shifted in such way, that the respective receive channel is free of disturbances. For the adaption, two different strategies are derived, analyzed and applied: Harmonics shifting and zero harmonics shifting. These strategies prove to be viable options that help to fulfill vehicle-internal EMC demands that mostly are much stricter than legal regulations and international standards.

Keywords - PWM, EMI, EMC, AM/FM Radio, Adaption

I. INTRODUCTION

The Pulse Width Modulation (PWM) is a common control scheme for electronic systems. Typical applications include e.g. DC-to-DC converters, electrical drives, electric heating and illumination. This scheme causes electromagnetic interferences (EMI) in a wide frequency spectrum [1]. The admissible EMI of a vehicle is limited by legal regulations [2] that are based on international standards [3]. Furthermore, there are vehicle-internal EMC demands that have to be considered in order to gain proper operation of all electronic systems. To fulfill these requirements, many different passive and active strategies have been developed and investigated in the past.

These solutions mostly aim at the reduction of disturbances in a wide frequency range. For the legal regulations, the broadcasting frequencies must be free from emissions in some distance from the source. As considered distances are quite large, legal regulations permit higher emission thresholds than the vehicle-internal demands.

Of course, the legal regulations have to be fulfilled in the whole frequency range. Considering the vehicle-internal EMC demands and an AM or FM radio as a typical EMI sink, the stricter limits must solely be applied to the current receive channel. This is the basic idea for the proposed techniques: Depending on the receive channel, the PWM frequency is adjusted so that there are no disturbances inside of the corresponding bandwidths. In [4], this idea is applied to mobile communication devices that utilize spread spectrum clocking.

This method requires a closed system in which the EMI source receives an information of the EMI victim. In Figure 1, an automotive application is depicted in which the radio announces the current receive channel to a disturbing converter

creating a feedback loop. After that, the converter adjusts its PWM frequency reducing its disturbances to the receive channel's bandwidth.





In [5], two basic strategies are patented that are both improved in this work. In the first strategy, the harmonics of a PWM signal are set equidistant around the known receive channel. This may cause a big alteration of the switching frequency. Thus, in this paper a milder but also sufficient condition is set. In [6], a similar approach is patented for the application in data systems that operate at a specific clock rate and additionally receive e.g. FM radio. In the second strategy, a harmonic is set onto the carrier frequency of the receive channel. For AM, the carrier may be so robust that the harmonic causes no distortion. Nevertheless, the carrier can be overdriven by a too large harmonic. Additionally, a beat develops if the harmonic and the carrier frequency are similar but not identical. As the reality is not ideal, it may not be possible to have identical frequencies for harmonic and receive channel. In this work, this strategy is improved in a way that the amplitude of the harmonic located on the carrier frequency is minimized. So, the possible disturbances are further reduced. Additionally, this improved method is extended to FM. For FM, there is no carrier but a center frequency. So, it is important that this frequency is free of disturbances as well. For both improved strategies, a theoretical approach is used that offers a clear analysis on the limitations and applicability.

This work has the following structure: At first, the spectrum of PWM signals is analyzed. From this analysis, two different strategies are derived to suppress disturbances inside of a chosen receive channel's bandwidth. Next, the assumed channels for AM and FM broadcasting are introduced. Afterward, for both strategies, detailed algorithms are presented that are applied to either AM or FM. To verify the results, measurements are done with a conventional DC-to-DC converter. In the conclusion, an outlook is given on possible applications.

II. THEORETICAL BACKGROUND ON PWM SIGNALS

An ideal PWM signal x(t) may be specified as shown in Figure 2 [1]. The signal has a switching frequency of f_{sw} and thus, a period of $T = 1/f_{sw}$. The duty cycle *d* specifies the width of the pulses by $\tau = d \cdot T$. x(t) has a maximum value of *A*.



Figure 2: Definition of a PWM signal

The coefficients of the Fourier series are defined as [1]:

$$|c_n| = A \cdot d \cdot |\operatorname{sinc}(n \cdot d)|$$
, $n \in \mathbb{N}$

So, there is a harmonic at every multiple of the switching frequency: $f_{\delta} = n \cdot f_{sw}$. Additionally, the sinc-function has a zero at every $f_{sinc,0} = m \cdot f_{sw}/d$, $m \in \mathbb{N}$ [1].

In Figure 3, an exemplary spectrum of a PWM signal with $f_{sw} = 100$ kHz, d = 0.25 and A = 48 V is shown. The signal could be the product of a DC-to-DC converter that connects the vehicular voltage levels 12 V and 48 V.



Figure 3: Calculated spectrum of a PWM signal

Firstly, the envelope of the sinc-function can be seen. Secondly, it is obvious that the amplitudes of the harmonics are the result of a sampling in the frequency spectrum. Thirdly, there are zero points for the harmonics at 400 kHz, 800 kHz and 1200 kHz. In this work, the harmonics with an amplitude of ideally 0 are called *zero harmonics* (zh). These zero harmonics are the result of a sampling at zero points of the sinc-function. So, there are zero harmonics for all spectral frequencies that satisfy $f_{sinc,0} = f_{\delta}$. The following relationship results:

$$\frac{m_{zh} \cdot f_{sw}}{d} = n_{zh} \cdot f_{sw} \Rightarrow d = \frac{m_{zh}}{n_{zh}} \text{ as reduced fraction}$$

So, every n_{zh} -th sample and every m_{zh} -th zero point of the sinc-function form a zero harmonic. In other words, every n_{zh} -th

harmonic is a zero harmonic having a weight of theoretically zero. The lower the number n_{zh} , the denser the zero harmonics become in the frequency spectrum. In this example, the following values apply: $d = 0.25 \Rightarrow m_{zh} = 1, n_{zh} = 4$. In Figure 3, it can be seen that every 4th harmonic and every 1st sinc-zero form a zero harmonic.

Two different strategies for disturbance suppression in receive channels are investigated in this work. For both strategies, the receive channel's bandwidths must to be free of harmonics. Only zero harmonics with an amplitude of approximately zero are tolerated inside of the channel's bandwidth.

The two strategies presented in this work are:

- 1. *Harmonics shifting*, the switching frequency is adapted to the receive channel so that there is no harmonic inside of the respective bandwidth.
- 2. *Zero harmonics shifting*, the zero harmonics are moved onto the carrier of the respective receive channel.

III. RECEIVE CHANNELS: AM AND FM BROADCASTING

In this work, broadcast systems according to Figure 4 are considered. For AM, f_{ch} is the carrier frequency. For FM, it is the center frequency. There is a signal bandwidth (*BW*) that carries the information. The goal is to eliminate all disturbances within the respective receive channel $f_{ch} - BW < f < f_{ch} + BW$.



Figure 4: Ideal receive channel

As an example, the European AM and FM broadcasting is investigated. AM has a frequency range from 526.5 kHz to 1606.5 kHz [7]. In Europe, the audio bandwidth is limited to 4.5 kHz. In the modulation scheme, this frequency range is mirrored around the carrier frequency. Thus, a bandwidth $BW_{AM} = 9$ kHz is needed for transmission [8].

European FM reaches from 87.5 MHz to 108 MHz [7]. The bandwidth of FM may be calculated by the Carson's Rule $BW_{FM} = 2(\Delta f + f_m)$ where Δf is the peak frequency deviation and f_m the highest frequency in the modulating signal. It states that 98 % of the signal power is inside of the calculated bandwidth. For a typical broadcast FM, the values $\Delta f = 75$ kHz and $f_m = 15$ kHz apply. So, the resulting bandwidth is about 180 kHz [9].

IV. HARMONICS SHIFTING

In the first technique, the harmonics of the PWM-spectrum are shifted by a variation of the switching frequency f_{sw} . The main goal is to move every harmonic out of the receive channel's bandwidth that reaches from $f_{ch} - \frac{BW}{2}$ to $f_{ch} + \frac{BW}{2}$. A secondary

criterion is to change the switching frequency as little as possible. Figure 5 shows an example in which the *n*-th harmonic (red) is inside of the receive channel's bandwidth. By a small alteration of f_{sw} , the critical harmonic is pushed out of the bandwidth (green).



Figure 5: Harmonics shifting

In [5], the receive channel has to be in between of two harmonics. This is a rather strict condition that may cause excessive alterations of the switching frequency. In the presented method, solely a distance of $\frac{BW}{2}$ to the carrier/center frequency has to be maintained reducing the necessary adaption drastically. For some cases in AM, it may be more convenient to set a harmonic onto the carrier as depicted in [5]. An improvement for this measure is discussed in V.

A. Algorithm

The following parameters are defined:

- Bandwidth of the receive channel: *BW*
- Nominal switching frequency: $f_{sw,nom} \ge BW$
- Carrier or center freq. of the receive channel: f_{ch}
- New switching frequency: $f_{sw} \ge BW$

There is the imperative condition of $f_{sw} \ge BW$. If f_{sw} is smaller, there would always be a harmonic inside of the receive channel.

In the following, the algorithm for harmonics shifting is described:

Firstly, the harmonic nearest to the receive channel is calculated by (nint is the **n**earest **int**eger function):

$$l = \operatorname{nint}\left\{\frac{f_{ch}}{f_{sw,nom}}\right\}$$

Secondly, three cases have to be considered: In the first case, the nearest harmonic is outside of the receive channel:

$$\left\{l \cdot f_{sw,nom} < f_{ch} - \frac{BW}{2}\right\} \lor \left\{l \cdot f_{sw,nom} > f_{ch} + \frac{BW}{2}\right\}$$

In this case, the receive channel has no disturbances. Thus, no shifting is necessary:

$$f_{sw} = f_{sw,nom}$$

In the second case, the nearest harmonic is inside of the receive channel and above/on the carrier/center frequency:

$$\left\{l \cdot f_{sw,nom} \ge f_{ch}\right\} \land \left\{l \cdot f_{sw,nom} < f_{ch} + \frac{BW}{2}\right\}$$

In order to use the smallest adaption possible, the nearest harmonic is increased to the upper boundary of the receive channel:

$$l \cdot f_{sw} = f_{ch} + \frac{BW}{2} \Rightarrow f_{sw} = \frac{2 \cdot f_{ch} + BW}{2 \cdot l}$$

In the third case, the nearest harmonic is inside the receive channel and below the carrier/center frequency:

$$\left\{l \cdot f_{sw,nom} < f_{ch}\right\} \land \left\{l \cdot f_{sw,nom} > f_{ch} - \frac{BW}{2}\right\}$$

Now, the nearest harmonic shall be decreased to the lower boundary of the receive channel:

$$l \cdot f_{sw} = f_{ch} - \frac{BW}{2} \Rightarrow f_{sw} = \frac{2 \cdot f_{ch} - BW}{2 \cdot l}$$

But this value is only feasible if $f_{sw} \ge BW$ is fulfilled. If not, an exception occurs and the nearest harmonic is to be set to the upper boundary again:

$$f_{sw} = \begin{cases} \frac{2 \cdot f_{ch} - BW}{2 \cdot l} & \text{if } \frac{2 \cdot f_{ch} - BW}{2 \cdot l} \ge BW\\ \frac{2 \cdot f_{ch} + BW}{2 \cdot l} & \text{else} \end{cases}$$

B. Application

As a demonstration, the technique is applied to a system with the following parameters: $f_{sw,nom} = 100$ kHz, AM: BW =9 kHz, 500 kHz $\leq f_{ch} \leq 1606.5$ kHz.

Figure 6 shows the result of the proposed algorithm. For most receive channels, no displacement is needed. Only for channels in the near of harmonics (500 kHz, 600 kHz, ...) the switching frequency has to be adapted. In this example, a relative shifting of merely ± 1 % is enough to disburden every receive channel from switching harmonics. Note that there is no dependency of the duty cycle. The dashed line shows the envelope that is valid for other switching frequencies as well.



Figure 6: Harmonics shifting for AM

Now, FM is investigated. It is obvious that the nominal switching frequency (100 kHz) is smaller than the channel's bandwidth (180 kHz). Thus, the switching frequency must be increased by 80 % to make harmonics shifting applicable. A more appropriate technique is needed for this case and will be presented in the following section.

V. ZERO HARMONICS SHIFTING

As explained in II, every n_{zh} -th harmonic of the PWM signal is a zero harmonic with an amplitude of ideally 0. In this strategy, the zero harmonics are shifted onto the carrier/center frequency of the receive channel. For an ideal zero harmonic, there are no disturbances in the receive channel's bandwidth if the condition $f_{sw} \ge \frac{BW}{2}$ is met. In Figure 7, an example is shown. Without zero harmonics shifting (red harmonics), there is a harmonic inside of the channel's bandwidth. By the application of zero harmonics shifting (green harmonics), the zero harmonic is moved onto the carrier/center frequency and the disturbing harmonic is shifted outside of the channel's bandwidth. In comparison to harmonics shifting, this method is feasible for smaller switching frequencies. In [5], any harmonic is set onto the carrier. In worst case scenarios, this may cause an overdrive or a beat frequency. This problem is avoided by the usage of zero harmonics.



Figure 7: Zero harmonics shifting

A. Algorithm

The following parameters are given:

- Bandwidth of the receive channel: BW
- Nominal switching frequency: $f_{sw,nom} \ge \frac{BW}{2}$
- Duty cycle: *d*
- Carrier or center freq. of the receive channel: f_{ch}
- New switching frequency: $f_{sw} \ge \frac{BW}{2}$

In this chapter, an algorithm is introduced to calculate the new switching frequency f_{sw} :

Firstly, the zero harmonics must be identified. According to II, every n_{zh} -th harmonic and every m_{zh} -th zero of the sinc-function result in a zero harmonic. The values are calculated by the reduced fraction of $d = m_{zh}/n_{zh}$. Thus, the zero harmonics are at frequencies of $f_{zh} = k \cdot n_{zh} \cdot f_{sw}$, $k \in \mathbb{N}$.

Secondly, the zero harmonic nearest to the receive channel has to be found. Note that k must be bigger than 0.

$$k \cdot n_{zh} \cdot f_{sw,nom} = f_{ch}$$

$$\Rightarrow k = \begin{cases} \operatorname{nint} \left\{ \frac{f_{ch}}{f_{sw,nom} \cdot n_{zh}} \right\} & \text{if } \operatorname{nint} \left\{ \frac{f_{ch}}{f_{sw,nom} \cdot n_{zh}} \right\} > 0 \\ 1 & \text{if } \operatorname{nint} \left\{ \frac{f_{ch}}{f_{sw,nom} \cdot n_{zh}} \right\} = 0 \end{cases}$$

So, the *k*-th zero harmonic is the closest to the receive channel. This is the $(k \cdot n_{zh})$ -th harmonic.

Thirdly, this harmonic needs to be shifted onto the carrier/center frequency:

$$\Rightarrow f_{sw} = \begin{cases} \frac{f_{ch}}{k \cdot n_{zh}} & \text{if } \frac{f_{ch}}{k \cdot n_{zh}} \ge \frac{BW}{2} \\ \frac{f_{ch}}{(k-1) \cdot n_{zh}} & \text{if } \frac{f_{ch}}{k \cdot n_{zh}} < \frac{BW}{2} \land (k-1) > 0 \\ \text{NaN} & \text{else} \end{cases}$$

The case distinction is due to the condition $f_{sw} \ge \frac{BW}{2}$. For $\frac{f_{ch}}{k \cdot n_{zh}} \ge \frac{BW}{2}$, the closest zero harmonic (k) is set onto the carrier/center frequency of the receive channel. If this condition is not met, the lower zero harmonic (k - 1) has to be chosen. This harmonic is then increased to the carrier/center frequency. If there is no lower harmonic, zero harmonic shifting is not applicable.

B. Influence of the duty cycle

There is a lower boundary for the applicability: The worst case occurs for the lowest possible nominal switching frequency $\left(f_{sw} = \frac{BW}{2}\right)$ and the usage of the first zero harmonic (k = 1). With these values, the lowest receive channel for which zero harmonics shifting is applicable may be calculated by:

$$k \cdot n_{zh} \cdot f_{sw} = f_{ch} \Rightarrow f_{ch,min} = n_{zh} \cdot \frac{BW}{2}$$

Under the assumption that $\frac{BW}{2}$ is an unchangeable constant, only n_{zh} influences $f_{ch,min}$. So, the duty cycle *d* constrains the frequency range indirectly.

The duty cycle is also most significant for the necessary adaption of the switching frequencies. As shown above, n_{zh} is defined by the reduced fraction of $d = m_{zh}/n_{zh}$. n_{zh} describes the "density" of zero harmonics in the frequency spectrum. The lower the number, the more zero harmonics exist. The more zero harmonics, the less shifting is needed. In Table 1, some duty cycles are considered. There are convenient values that cause small values n_{zh} . On the other hand, there are disadvantageous duty cycles that cause high values of n_{zh} making zero harmonics sparse in the frequency spectrum.

Table 1: Exemplary duty cycles

d	m_{zh}/n_{zh}	n _{zh}
50 %	1/2	2
33, 3 %, 66, 6 %	1/3,2/3	3
25 %, 75 %	1/4,3/4	4
20 %, 40 %, 60 %, 80 %	1/5,2/5,3/5,4/5	5
10 %, 30 %, 70 %, 90 %	1/10,3/10,7/10,9/10	10
5 %, 15 %, 35 %,	1/20,3/20,7/20,	20
1 %, 3 %, 7 %,	1/100,3/100,7/100,	100

C. Application

As a demonstration, the technique is applied to a system with the following parameters: $f_{sw,nom} = 100$ kHz, FM: BW =180 kHz, 87.5 MHz to 108 MHz. Due to the wide bandwidth, simple harmonics shifting is not feasible anymore. So, zero harmonics shifting is necessary. To get an impression of the influence of the duty cycle, three different n_{zh} are considered. The result of the algorithm is depicted in Figure 8. It can be seen that lower values of n_{zh} cause a low necessary alteration of the switching frequencies. Higher values of n_{zh} require higher variations of the switching frequency. Nevertheless, even for $n_{zh} = 100$, a relative shifting of approximately ± 5 % is enough to make every respective FM channel free of disturbances. In comparison, harmonics shifting would immediately force a relative shifting of +80 % due to its requirement of $f_{sw} \geq BW$.



Figure 8: Zero harmonics shifting for FM

D. Solutions for disadvantageous duty cycles

This part discusses possible solutions for duty cycles that require major changes in switching frequency or fail due to the limitations analyzed above. The applicability has to be considered against the background of the system.

One solution is an additional tuning of the duty cycle. There are two strategies that can be pursued. In the first, disadvantageous duty cycles are forbidden. Therefore, the nearest acceptable duty cycle is chosen. With this new duty cycle, the above algorithm is applied again. In the second strategy, an alternative algorithm is used:

- 1. Calculation of the nearest harmonic: $n = nint \left\{ \frac{f_{ch}}{f_{sw,nom}} \right\}$
- 2. Calculation of the switching frequency: $f_{sw} = \frac{f_{ch}}{n}$
- 3. Calc. of the nearest sinc-zero: $m = nint \left\{ d_{nom} \cdot \frac{f_{ch}}{f_{sw}} \right\}$
- 4. Calculation of the new duty cycle: $d = \frac{m}{n}$

Another solution uses an alternating scheme of PWM signals with applicable duty cycles d_i , resulting switching frequencies $f_{sw,i}(d_i)$ and durations of T_i . The values of T_i have to be set in such way that the time average delivers the original duty cycle:

$$d_{original} = \frac{1}{\sum T_i} \cdot \sum d_i \cdot T_i$$

VI. MEASUREMENTS

To verify harmonics shifting and zero harmonics shifting, both techniques are applied to a test setup.

A. Test setup

Figure 9 shows the topology of the test setup. A vehicular 12 V battery is the power source for a DC-to-DC converter that boosts the voltage for a 27.2 Ω load resistor. Conducted emissions in the AM range are measured with the usage of an artificial network. For the measurement of radiated emissions in FM range, the converter is placed inside of a TEM cell. The converter operates at a nominal switching frequency of 100 kHz. The PWM signal of U_{sw} is depicted in Figure 10. Its duty cycle is set to 25 %. Obviously, both system and PWM signal are non-ideal.



Figure 10: PWM signal of the boost converter

B. Harmonics shifting for AM

To verify harmonics shifting, the conducted EMI (U_{AN}) is measured with an artificial network and a RBW of 200 Hz. It is assumed that the current AM receive channel is at a frequency of 1000 kHz. The spectra are shown in Figure 11.



Figure 11: Measurement for harmonics shifting

Without harmonics shifting ($f_{sw,nom} = 100$ kHz), one harmonic is exactly in the center of the channel's bandwidth. By an alteration of the switching frequency to 99.55 kHz, the harmonic is shifted to the boundary of the receive channel. So, the disturbances are successfully suppressed if the switching frequency is reduced by only -0.45 %.

C. Zero harmonics shifting for FM

To evaluate zero harmonics shifting, the radiated emissions are measured by the usage of a TEM cell. As an example, a FM channel at 101 MHz is assumed as the receive channel. The spectra depicted in Figure 12 are measured with a RBW of 9 kHz. Without zero harmonics shifting ($f_{sw,nom} = 100$ kHz), there is a harmonic in the center of the channel's bandwidth. With zero harmonics shifting ($f_{sw} = 100.2$ kHz), the lower zero harmonic is shifted to the center frequency. By a switching frequency adaption of just +0.2 %, the spectral component at the center frequency is effectively reduced by approximately 23 dB μ V.



Figure 12: Measurement for zero harmonics shifting

VII. CONCLUSION

In this work, the PWM frequency of an EMI source is dynamically adapted so that a specific receive channel (for e.g. AM or FM broadcasting in automobiles) is undisturbed.

At first, the frequency spectrum of a PWM signal is discussed. From characteristic features, two different strategies are derived: Harmonics shifting and the zero harmonics shifting.

In harmonics shifting, the empty frequency ranges between harmonics are used. The method is easy to implement and independent of the duty cycle. Nevertheless, it is only feasible for switching frequencies that are at least as high as the receive channel's bandwidth. Vehicular control units usually use PWM signals around 20 kHz in order to operate e.g. actuators or heating elements. This frequency is much higher than the AMbandwidth of 9 kHz. Power electronic devices such as on-board chargers often use switching frequencies around 100 kHz. For AM, harmonics shifting is applicable. For FM and its respective bandwidth of approximately 180 kHz, this method is not suitable. If switching frequencies higher than 180 kHz are implemented, harmonics shifting is applicable for FM again. Due to the independency of the duty cycle, harmonics shifting may be easily applied to systems utilizing a varying duty cycle such as e.g. power factor corrections or motor inverters.

In zero harmonics shifting, harmonics with an amplitude of theoretically zero are utilized. In comparison to harmonics shifting, this method is applicable for switching frequencies that are only half of the receive channel's bandwidth. The method is dependent on the duty cycle. Hence, the implementation is more complex and there are more limitations than for harmonics shifting. Nevertheless, it may be applied to FM and DC-to-DC converters that utilize switching frequencies around 100 kHz because these frequencies are more than half of the assumed FM bandwidth of 180 kHz. With additional effort, this strategy may also be used for power electronic systems with widely changing duty cycles. Furthermore, zero harmonics shifting may be applied to AM and control units that utilize frequencies as little as 4.5 kHz.

As shown by measurements, harmonics shifting and zero harmonics shifting are viable techniques to meet vehicle-internal demands on EMC. Except the information line from the radio to EMI source, there is no additional component expense as only the switching frequency of the device has to be adjusted. By applying either harmonics shifting or zero harmonics shifting, the respective receive channel in AM or FM is ideally completely free of disturbances.

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